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ARRAYS OF HEATED HORIZONTAL CYLINDERS IN NATURAL CONVECTION

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Abstract—The heat transfer properties of a vertical array of heated cylinders are obtained in steady state natural convection. The results for a variety of combinations of spacing and number of cylinders are presented. For each of these combinations a variety of dissipation rates have been employed, yielding cylinder Grashof numbers from 750 to 2000. The results are displayed in terms of cylinder surface temperature excess, normalized with respect to the temperature excess of the lowest cylinder. As in the case of the studies of Lieberman and Gebhart [2], the surface temperature increases with elevation in the array for closely spaced arrays, but decreases with elevation when the spacing is sufficiently large. This behaviour is explained in terms of the velocity and temperature fields in the wake of a line source. Agreement between the present results and those of reference [2] is shown to be satisfactory when the correct length is used in the Grashof number. Because of the mutual interaction of the cylinders, the cylinder Nusselt numbers are not predicted by the usual natural convection correlations. Actual cylinder Nusselt numbers are compared with those predicted for simple natural convection at the cylinder Rayleigh number. This ratio is plotted with respect to position in the array. Average Nusselt numbers for the arrays are also plotted with respect to the Rayleigh number.

NOMENCLATURE

A, area of a single cylinder;
A_T, total heat transfer surface area;
B, coefficient in equation (4);
B_{ij}, radiation absorption factors;
C_1, C_2, constants in wake calculations;
C_3, C_4, C_5, C_6, influence coefficients in wake calculations;
D, cylinder diameter;
F_{ij}, radiant exchange view factor between surfaces i and j;
I, heating current;
K, factor in radiation correction calculation;
L, length of heated cylinder between voltage taps;
Nu, Nusselt number $(h_c D/k)$;
Nu_i, Nusselt number of ith element;
Nu_{ave}, average Nusselt number for the array;
Nu_0, Nusselt number for lower cylinder in wake calculation;
$N_{G,D}$, Grashof number based on cylinder diameter, $(g\beta\Delta T D^3)/\nu^2$;
$N_{G,X}$, Grashof number based on position in array $[N_{G,D}(X/D)^3]$;
N_{RA}, Rayleigh number $(N_{G,D} \cdot N_{PR})$;
N_{PR}, Prandtl number;
Q, dissipation rate;
Re, Reynolds number;
S, spacing, in cylinder diameters;
T_S, surface temperature;
T_∞, ambient air temperature;
T_e, wake centreline temperature;
ΔT, temperature excess $(T_s - T_\infty)$;
U_T, overall heat transfer coefficient;
V, voltage drop across heated element;
X, distance upward in array from bottom cylinder;
h, apparent surface heat transfer coefficient;
h_c, convective heat transfer coefficient;
h_r, radiative surface heat transfer coefficient;
h'_r, corrected radiative surface heat transfer coefficient;
k, conductivity of air;

n, exponent in equation (4);
u_e, wake centreline velocity;
x, coordinate along wake centreline;
β, coefficient of expansion of air;
ε, surface emissivity;
Φ, normalized temperature excess, $(T_s - T_\infty)/(T_{s_1} - T_\infty)$;
σ, radiation constant;
ν, kinematic viscosity of air.

Subscripts
i, j, cylinder identification;
0, refers to bottom cylinder in wake calculation.

INTRODUCTION

A GOOD deal has been written about the heat transfer characteristics of cylinders in natural convection. In almost all of the investigations, attention has been restricted to consideration of a single cylinder. Only a few investigators [1, 2] have attempted to study interactions between two or more horizontal cylinders. Yet arrays of cylinders (tubes) which exchange heat via natural convection are commonplace: such arrays are frequently used (with, perhaps, some extended surface modifications) as condensers in domestic refrigeration apparatus. Also, heaters for high viscosity fuel oils may consist of arrays of heated tubes [3].

A number of years ago, Eckert and Soehngen [1] carried out a limited study of heated cylinders which interacted in natural convection. Three cylinders were arranged in a vertical array, and the measured Nusselt numbers showed that the upper cylinders exhibited increasingly higher temperatures. When the three cylinders were arranged in a staggered array they found that the Nusselt number for the offset cylinder was higher than that for the bottom cylinder, while the Nusselt number of the top cylinder (aligned with the bottom one) was less than that of the bottom cylinder. In these experiments solid copper cylinders were used; the length to diameter ratio was about 13 and the Grashof number (based on cylinder diameter) was 34300 for the vertical array and 14650 for the staggered array.

Recently, Lieberman and Gebhart [2] have conducted experiments on the interactions of heated wires arranged in a plane array. The wires were 0·184 m long and 0·127 mm dia. The array could be oriented so that its plane made angles of 0°, 30°, 60° and 90° with the vertical. The heated wire spacing could be varied from 37·5 diameters to 225 diameters. This arrangement provided data for Grashof numbers of the order of 10^{-1}. Moreover, the wire spacings were much larger than might be encountered in practical heat exchangers.

The evident lack of experimental data for interactions in natural convection prompted the present study. This work deals with plane arrays of cylinders heated in air. Cylinder dimensions are such that realistic Grashof numbers are attained. The apparatus may be adapted to studies in other fluids and with other effects present, such as acoustic and vibrational effects.

DESCRIPTION OF EXPERIMENTAL APPARATUS AND MEASUREMENTS

General

The apparatus consists of an array of cylinders (tubes), mounted with their axes horizontal, and in a single plane. The cylindrical elements are made of thin walled (0·152 mm) stainless steel tubing, 6·35 mm o.d. These tubes are arranged in series electrically and are resistance heated. The tubes are 58·5 cm long; the electrical resistance of each is about 0·105 Ω.

Array support assembly

The cylinders are clamped between two support rods which are in turn bolted to a horizontal cross member. The vertical support rods are 19·1 mm dia. Each rod was drilled with a series of 6·35 mm holes, and then sawed along the line of centres of these holes to form the clamping device. The arrangement is shown in Fig. 1. The holes in the support rod are arranged so that various spacings could be achieved,

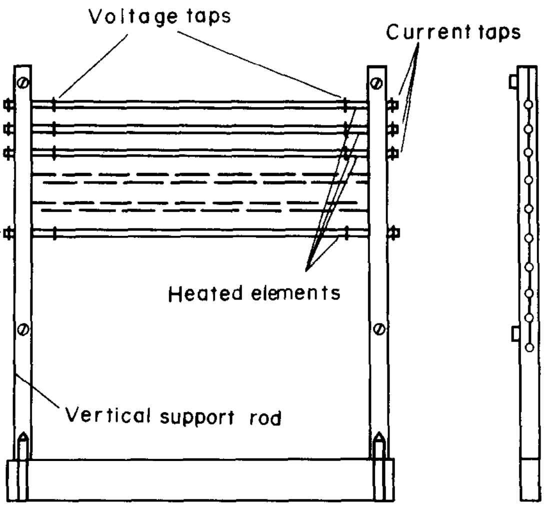

FIG. 1. Arrangement of heated cylinders in vertical array.

with a minimum spacing of 2 diameters. For fewer tubes larger spacings can be used. The heated cylinders are thermally and electrically insulated from the support rods.

Heated cylinders

The thin walled cylinders are fitted with current and voltage connections which are simply rings of 16 gauge copper wire silver soldered circumferentially around the tubes at appropriate locations. These provide good electrical contact at well defined locations on the tubes. The current connections are outboard of the support rods, near the ends of the tubes. The voltage connections are inboard of the support rods, and are 44·5 cm apart. In this way, the tubes are heated for their entire length, but voltage measurements are made only over the central portion of the cylinder. This arrangement reduces end losses by providing an effective guard ring at each end of the heated cylinder. An axial temperature profile for a single tube is shown in Fig. 2. Near one end, at the location of the voltage tap, the axial temperature gradient is about 583 K/m. The axial heat conduction for such a gradient is found to be approximately 0·0293 W. The dissipation rate used for this profile was 9·95 W.

Thus the end loss (assuming both ends are the same) is less than 1 per cent of the total dissipation. In view of this, end losses have been neglected.

The voltage connections are brought out to a ten-position switch and thence to a voltmeter, so that the voltage drop across each test section can be measured. Because the tubes are in series, a single current measurement suffices to determine the dissipation rate.

Temperature sensing

A single copper–constantan thermocouple (30 gauge) located at the mid-point of the active length of each tube is used to sense surface temperature. Because of the low heat transfer coefficients and the relatively high conductivity of the tube material, circumferential temperature gradients are expected to be small. Attempts to detect circumferential variations by rotating the thermocouple within the cylinder have been unsuccessful; the circumferential temperature is constant to within the accuracy of the experiment.

The thermocouple junction is passed through

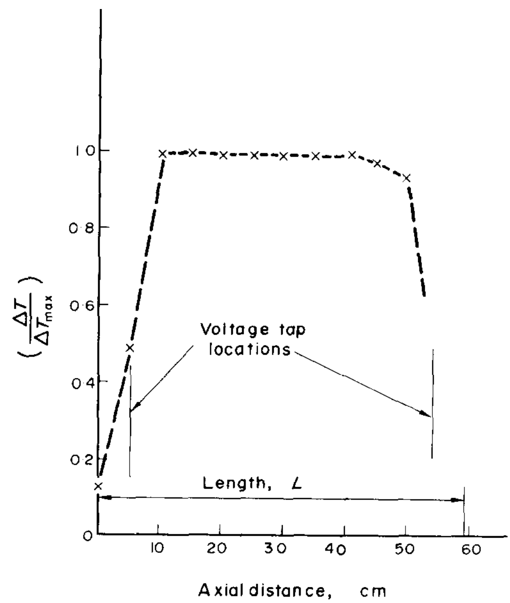

FIG. 2. Axial temperature distribution in heated cylinder.

a hole drilled axially in a 6·35 mm dia. teflon cylinder which is 6·35 mm long. The teflon cylinder fits tightly into the heated tube. Before the teflon cylinder is pushed into a tube, the thermocouple junction is folded back so that it is squeezed between the teflon and the tube wall. The arrangement is shown schematically in Fig. 3. The conductivity of teflon is sufficiently low so that its presence in the tube has negligible influence on the local temperature of the tube. The thermocouple junction also may affect the electrical properties locally, but this is minimized by keeping the junctions small.

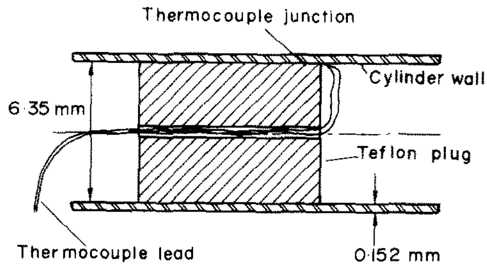

FIG. 3. Detail of mounting thermocouple junctions.

Mounting

The support rods and horizontal cross member are mounted on a compound table, the face of which is vertical. Thus the plane of the array may be rotated to make any angle with the vertical. In addition the assembly may be completely inverted to permit checking for asymmetry effects (none have been found).

Because of air movements in the laboratory, the entire array is housed within an enclosure 1·219 m × 1·219 m × 2·438 m high. A small gap between the floor and the enclosure walls permits limited air circulation; the top of the enclosure is open.

MEASUREMENTS AND OBSERVATIONS

Heat transfer parameters have been obtained for the configurations listed on Table 1. The following paragraphs outline the measurements and observations made.

The voltage drop across each tube is measured several times during the observation of the thermocouple outputs. Fluctuations in voltage are small, but there are small variations (±3 per cent) in the resistance of the tubes. A Hewlett–Packard 419A Null Voltmeter is used for voltage measurements.

The thermocouple outputs have been measured in two ways. Initially, a Rubicon potentiometer was used, yielding readings accurate to within 10 μV. These observations were taken over a sufficiently long period (about 15 min) to ensure that the steady state was attained. A number of readings were taken after the steady state was reached. These data were then averaged, and the standard deviation was found to be generally less than 1 per cent of the mean for any set of readings. The reference junction was maintained at 273 K in an ice/water bath. Latterly, a Honeywell potentiometric multipoint recorded has been used, providing a record of the temperatures at each cylinder and a reference temperature. Again, these records are averaged over a period of time following the attainment of the steady state. The standard deviation of these sets of data has been found to be about 1 per cent of the mean.

Ambient temperature is observed by means of a thermocouple mounted below the heated elements.

Because direct current is used for heating, it is necessary to reverse the current leads during each set of observations to avoid the error caused by the voltage drop across the thermocouple junctions [4]. Thus for each dissipation rate, two sets of data are analyzed. The temperatures corresponding to the averages of the two sets of thermocouple readings are averaged. In the event that the temperatures differ significantly the error is taken to be one half the difference between the temperatures corresponding to the means. In some cases, the error resulting from this is as much as 5 per cent.

Heating power is supplied by a Hewlett–Packard Harrison 6434B power supply. The current is measured using a d'Arsonval ammeter.

At the beginning of the study, measurements were made on single cylinders to (i) check the longitudinal temperature profile, and (ii) to check that the natural convection results agreed with the published literature [5, 6]. Upon taking radiation into account, agreement with published correlations is good. Because the tubes were heated to a fairly high temperature (up to 505 K) the resulting surface emissivity is difficult to determine precisely. Thus the radiation effect is hard to assess. A sampling of tabulated values of emissivity indicated that values from about 0·2 up to about 0·6 might be appropriate. The data were reduced using a value of 0·3 for the emissivity. Because the radiative surface coefficient is never more than 35 per cent of the overall surface coefficient, the resulting error in the convection coefficient would be about 14 per cent for emissivity values of 0·2 or 0·4. However, the good agreement with published data suggests that the emissivity value chosen is reasonable.

Perhaps more important is the radiative interchange between the cylinders. The data have been reduced using an approximate correction for the radiation between adjacent cylinders. The enclosure is assumed to be at the ambient temperature. For very close spacing, the view factor between tubes becomes significant and the radiative loss from the tubes is reduced. The effect of radiative exchange with adjacent tubes is discussed in the Appendix.

RESULTS

Vertical arrays of heated cylinders have been observed in a variety of configurations. The arrangements for which data are reported are listed in Table 1 below.

The temperature excess for each cylinder, normalized with respect to the temperature excess for the bottom cylinder is shown in Figs. 4–6 for the arrays of three, five and nine tubes respectively. The Grashof number range for these figures is from 750 to 2000. The spacing and heating currents are shown on each figure. Figure 7 is a plot of data taken by Hsieh [7], using a similar geometry but different heating and measurement techniques. The similarity of the temperature distribution in Figs. 5 and 7 is evident.

The heat transfer capability of each cylinder is characterized by its Nusselt number. For natural convection, this is a function of the Rayleigh number. In Figs. 8 and 9, the Nusselt numbers of each cylinder and at each spacing are plotted. The cylinder Nusselt number, Nu_i,

Table 1. Tabulation of configurations and dissipation rates
(plane of array vertical in all cases)

No. of cylinders	Heating current (A) and corresponding nominal dissipation rate (W/m²)					Spacing (dia.)
3	6	9	12	15	(A)	2
	433	993	1830	2980	(W/m²)	4
						6
						10
						20
5	6	7·5	9	12	15	2
	438	698	1050	1890	3120	4
						6
						10
9	6	7·5	9	10·5		
	431	688	1030	1430		2
	429	678	1005	1860		4
	429	678	1005	1850		6
	429	678	994	1840		10

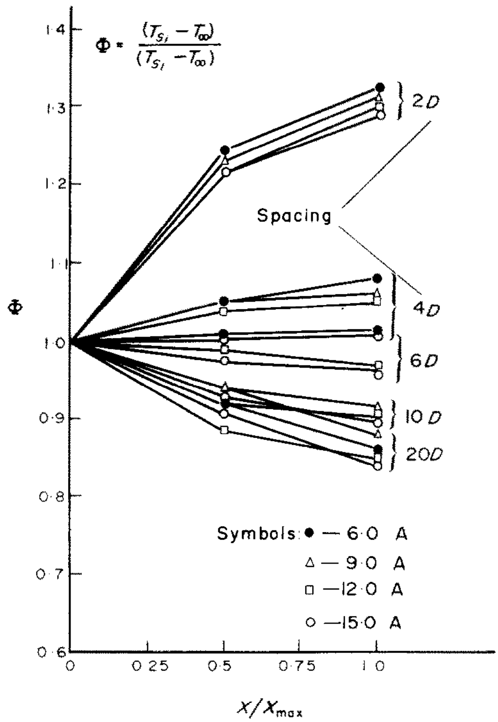

FIG. 4. Temperature distribution in three-cylinder array.

has been normalized with respect to the Nusselt number *for a single cylinder*, Nu_0, at that Rayleigh number. Thus these plots compare observed Nusselt number with predicted Nusselt number for a single cylinder. It is clear from these figures that the bottom cylinder behaves almost as if the other cylinders were not present. Its heat transfer capability is enhanced at large spacings, which is believed to be due to the flow induced by the entire array. The upper cylinders display reduced Nusselt numbers at close spacing and enhanced Nusselt numbers at large spacing.

DISCUSSION

Referring to the temperature excess plots (Figs.4–7) it is clear that the temperature rises, as one would expect, with elevation in the array, for closely spaced arrays. However, the 9-tube array shows a departure from this behaviour at the top tube, even for the closest spacing. A dropoff in temperature is observed at the top tube. In wider spacings, for all arrays, a dropoff is observed, and for the widest spacings the temperature drops monotonically with elevation in the array. This behaviour is explained in the following paragraphs.

First, consider the monotonic decrease in the widely spaced case. Confining attention to the bottom two tubes, it is noted that for sufficient separation, the upper tube lies in the "far wake" of the lower tube. Now in the far wake, the details of the size and shape of the lower tube are unimportant. Indeed, that is what is implied by the "far wake". In this case, the bottom cylinder may be replaced by an "effective line source" of heating. (The position of the effective line source may not coincide with the axis of the real cylinder.) For a line source the similarity solution shows [8] that the velocity and temperature fields along

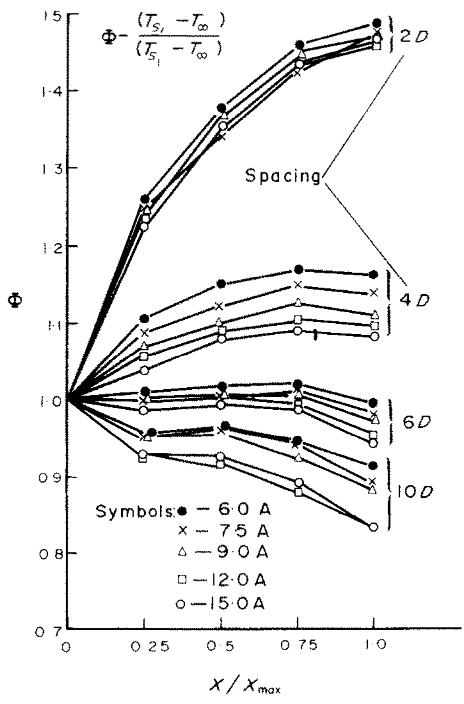

FIG. 5. Temperature distribution in five-cylinder array.

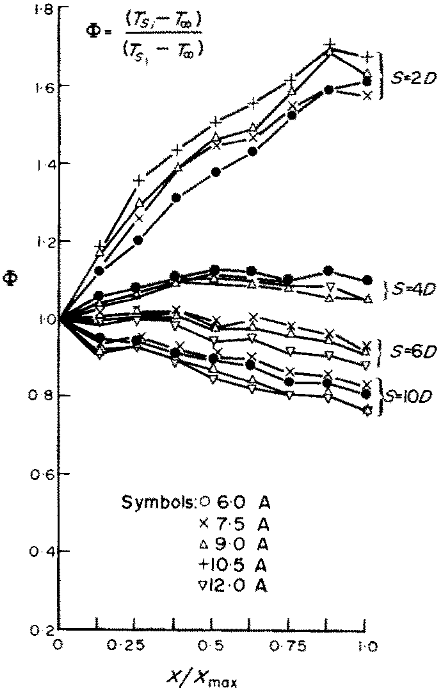

FIG. 6. Temperature distribution in nine-cylinder array.

the wake centre-line are given by:

$$u_e \propto Q^{\frac{2}{5}} x^{\frac{1}{5}} \tag{1}$$

$$(T_e - T_\infty) \propto Q^{\frac{4}{5}} x^{-\frac{3}{5}}. \tag{2}$$

Now referring to Fig. 10, we write,

$$(T_s - T_e) = Q_1/(kN_{u_0}\pi L) \tag{3}$$

$$u_e = C_1 Q_0^{\frac{2}{5}} x^{\frac{1}{5}} \tag{1a}$$

$$(T_e - T_\infty) = C_2 Q_0^{\frac{4}{5}} x^{-\frac{3}{5}}. \tag{2a}$$

These relations hold along the centreline of the wake, far from the lower cylinder and also assume that the wake is uninfluenced by the presence of the upper cylinder.

The upper cylinder is exposed to a velocity u_e; thus it is no longer in pure natural convection. Indeed, we need to evaluate a Reynolds number for the cylinder. For a 6·35 mm diameter cylinder in air, and $Q_0 = 48$ W/m, this Reynolds number is between 80 and 150 for spacings from 2 to 20 diameters. The Nusselt number is given by [9].

$$Nu = BRe^n \tag{4}$$

where $B = 0{\cdot}48$ and $n = 0{\cdot}51$. The temperature loading effect has been neglected.

Now,

$$(T_s - T_\infty) = (T_s - T_e) + (T_e - T_\infty)$$

therefore,

$$(T_s - T_\infty) = C_3 Q_1/(k\pi L B Re^n) + C_2 C_4 Q_0^{\frac{4}{5}} x^{-\frac{3}{5}}.$$

Here, C_3 and C_4 are factors which reflect the influence of the upper cylinder. For a single cylinder,

$$(T_{s_0} - T_\infty) = Q_0/h_0 A.$$

If the presence of an upper cylinder is represented

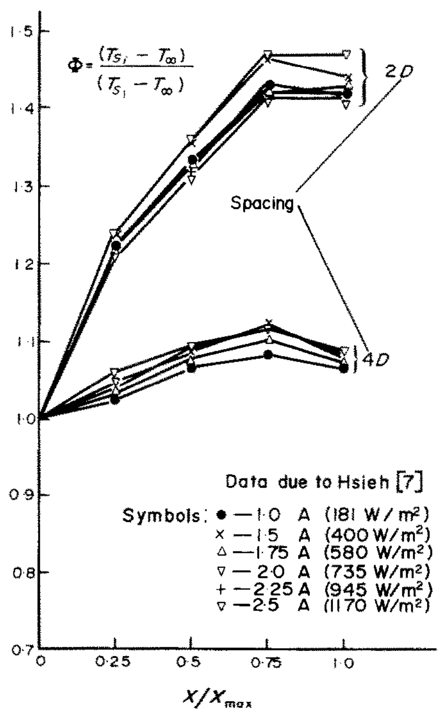

FIG. 7. Temperature distribution in five-cylinder array. (Data by Hsieh[7].)

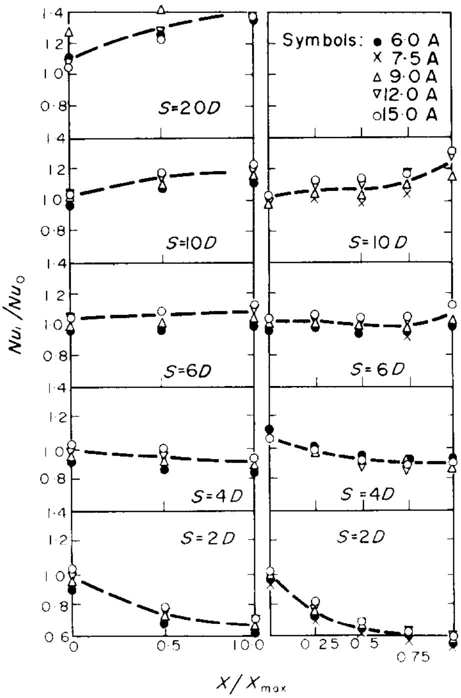

FIG. 8. Nusselt number distribution for three- and five-cylinder arrays. Nusselt number is normalized with respect to Nusselt number predicted for natural convection for a single cylinder at the Rayleigh number of that cylinder.

in this case by an influence coefficient C_6, we may write for the lower cylinder,

$$(T_{s_0} - T_\infty) = C_6 Q_0 / h_0 A. \quad (5)$$

Applying an influence coefficient C_5 to the expression (1a) for u_e, the temperature excess ratio, Φ, is:

$$\Phi = \frac{(T_s - T_\infty)}{(T_{s_0} - T_\infty)}$$

$$\Phi = \left(\frac{C_2}{C_6}\right)\frac{C_4 h_0 A}{Q_0^{\frac{1}{4}} x^{\frac{3}{4}}} \times \left[1 + \left(\frac{Q_1}{Q_0}\right)\left(\frac{C_3}{C_4}\right)\frac{Q_0^{(1-2n)/5}\, x^{(3-n)/5}\, \nu^n}{C_2\, k\, BAD^{n-1}(C_1 C_5)^n}\right]. \quad (6)$$

Thus, for large values of x, the temperature excess ratio is proportional to $x^{-0.102}$. The foregoing argument applies only to the bottom two cylinders. Proceeding up the array, each succeeding cylinder is embedded in a temperature and velocity field where the velocity effect on the convection coefficient outweighs the effects of the increased "ambient" temperature.

As the cylinder spacing is reduced, the upper cylinder in any pair is increasingly affected by the details of the near wake, and the preceding arguments do not apply. Radiative exchange is also more significant, and at present it does not seem possible to predict the "ambient" conditions satisfactorily.

Returning again to Fig. 6, it is noted that the upper tube in the 9-tube array is cooler than the one below it, which seems remarkable. But considering the other temperature excess plots, it is observed that the falloff occurs in the 5-tube array at the 4-diameter spacing, and in the 3-tube array at the 6-diameter spacing. This implies that the characteristic length of the

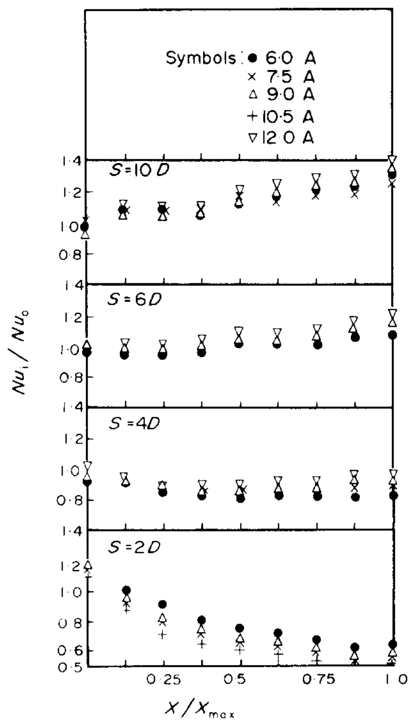

FIG. 9. Nusselt number distribution for nine-cylinder array. (Refer to description of Fig. 8.)

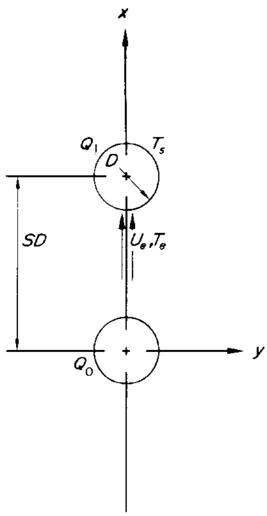

FIG. 10. Diagram of wake–cylinder interaction.

problem is not the tube diameter, but rather the position in the array. Forming the Grashof number for these tubes using the distance measured from the bottom tube as the characteristic length, yields:

for tube #9 at 2 diameters,

$$N_{G,x} = N_{G,D}(2 \times 8)^3 \doteq 4100 N_{G,D}$$

for tube #5 at 4 diameters,

$$N_{G,x} = N_{G,D}(4 \times 4)^3 \doteq 4100 N_{G,D}$$

for tube #3 at 6 diameters,

$$N_{G,x} = N_{G,D}(6 \times 2)^3 \doteq 1700 N_{G,D}.$$

In each case the x-based Grashof number is a few thousand times greater than the diameter based Grashof number. The actual x-based Grashof numbers for the above cases range from 2×10^6 to 8×10^6. The results of Lieberman and Gebhart [2], indicate a dropoff for the 112·5 diameter spacing case. Here the x-based Grashof number of the top wire is

$$N_{G,x} = 1{\cdot}75 \times 10^{-2} \times (3 \times 112{\cdot}5)^3 \doteq 6{\cdot}7 \times 10^5.$$

This suggests that vertical arrays of interacting cylinders undergo some sort of transition at Grashof numbers (based on position) of the order of 10^6–10^7. In other words, the observed temperature excess behaviour is adequately explained in terms of a Grashof number in which the characteristic length dimension is the distance from the bottom element to the cylinder in question, rather than simply the diameter of the cylinder. Clearly more experimental evidence is needed to verify this conclusion.

While the foregoing arguments deal with the temperature distribution through the vertical array, the designer of heat exchangers needs information about the gross heat transfer characteristics of such an array. An overall heat transfer coefficient, U_T, is defined as follows

$$Q = U_T A_T \overline{\Delta T}$$

where Q is the total electrical dissipation, A_T is the total heat transfer surface area and $\overline{\Delta T}$ is an average temperature excess for the array.

Now $\quad A_T = NA$

and $\quad Q = I \sum_{i=l}^{N} V_i.$

Defining $\quad \overline{\Delta T} = (\sum_{i=l}^{N} \Delta T_i)/N$

then $\quad U_T = NQ/(A_T \sum_{i=l}^{N} \Delta T_i)$

or
$$U_T = \frac{NQ}{A_T \Delta T_1} \frac{1}{\sum_{i=l}^{N} \Phi_i}. \tag{7}$$

Thus, with data such as that shown in Figs. 5–8 available, appropriate spacings may be found. The evidence so far indicates large U_T for large spacings. However, large spacings imply large overall dimensions. Thus, optimization consists of examining a trade-off between spacing and overall size of the heat exchanger. Optimization studies have not yet been carried out.

Figures 11–13 illustrate the average Nusselt number behaviour plotted against the Rayleigh

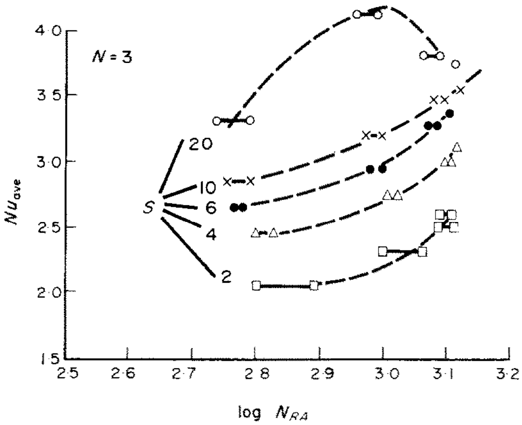

FIG. 11. Average Nusselt number for three-cylinder array vs. Rayleigh number range for array. (---: faired curve).

number range for a particular dissipation rate. The dashed lines indicate the trends of the data. The increase in Nusselt number with increased spacing is clearly evident.

ERRORS

Measurements in natural convection are rarely easy to perform with great accuracy. These experiments are no exception. While individual thermocouple measurements can easily be taken to less than 1 per cent, time variations of the cylinder temperatures are much larger than this. Furthermore, the effects of d.c. heating on a thermocouple in electrical contact with the cylinders results in an error. These errors are reduced by time averaging and averaging results for opposing current directions. In the most severe cases, variations of 5 per cent in surface temperatures have been found.

Heating rates are fairly accurately determined, with voltage and current readings characteristically better than 1 per cent. However, dissipation rates are not identical for all cylinders due to minor variations in the resistivity of the cylinder material. Thus, Figs. 4–6 contain small (usually about 1 per cent) variations in heating power. This accounts for some of the considerable scatter which is especially evident at the higher heating rates.

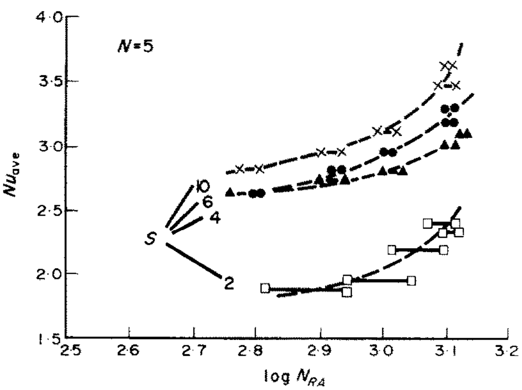

FIG. 12. Average Nusselt number for five-cylinder array vs. Rayleigh number range for array. (---: faired curve).

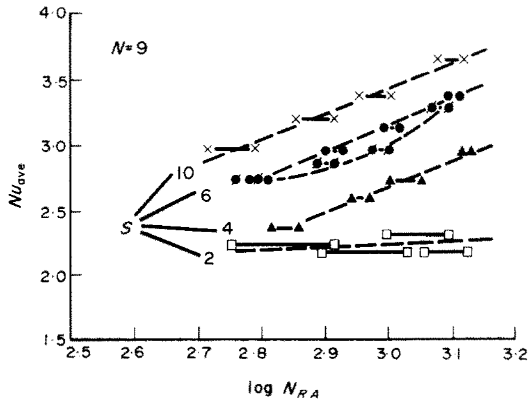

FIG. 13. Average Nusselt number for nine-cylinder array vs. Rayleigh number range for array. (---: faired curve).

CONCLUSIONS

The heat transfer characteristics exhibited by vertical arrays of heated horizontal cylinders are not predicted by simple superposition of single cylinder behaviour. For closely spaced arrays, individual tube Nusselt numbers are found to be smaller than for a single cylinder (as much as 50 per cent smaller). However, for wide spacings, individual tube Nusselt numbers are higher (up to about 30 per cent) than for a single cylinder. Thus the overall heat transfer characteristics of an array are dependent upon array spacing as well as Rayleigh number. Furthermore, tube Nusselt numbers are dependent upon position in the spacing. The bottom tube in any array behaves much like a single tube; the upper tubes show significant differences.

The cylinder temperature in a vertical array is a function of spacing and position for a given dissipation rate. For close spacings, the temperature rises, generally, but shows a fall-off if the Grashof number based on cylinder distance from the bottom of the array exceeds a certain critical value. For wide spacings the temperature decreases monotonically. This behaviour is consistent with the properties of wakes above line sources of heat.

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REFERENCES

1. E. G. Eckert and E. Soehngen, Studies of heat transfer in laminar free convection with the Zehnder–Mach interferometer, Tech. Report No. 5747 USAF AMC Dayton, Ohio (1948).
2. J. Lieberman and B. Gebhart, Interactions in natural convection from an array of heated elements, experimental, *Int. J. Heat Mass Transfer* **12**, 1385–1396 (1969).
3. J. F. Corpe, The heating of high viscosity oils by natural convection, Fourth Int. Heat Transfer Conf. Paper NC 4-4, Paris-Versailles (1970).
4. M. E. Davenport, P. M. Magee and G. Leppert, Thermocouple attachment to a direct current heater, *J. Heat Transfer* **84**, 187–188 (1962).
5. B. G. Van der Hegge Zijnen, Modified correlation formulae for the heat transfer by natural and by forced convection from horizontal cylinders, *Appl. Sci. Res.* **6A**, 129–140 (1957).
6. W. H. MacAdams, *Heat Transmission*, 3rd Ed. McGraw-Hill, New York (1954).
7. R. Hsieh, private communication (Master's Thesis in preparation).
8. B. Gebhart, L. Pera and A. W. Schorr, Steady laminar natural convection plumes above a horizontal line heat source, *Int. J. Heat Mass Transfer* **13**, 161–171 (1970).
9. D. C. Collis and M. J. Williams, Two dimensional convection from heated wires at low Reynolds numbers, *J. Fluid Mech.* **6**, 357–384 (1959).
10. B. Gebhart, *Heat Transfer*. McGraw-Hill. New York (1959).
11. G. F. Marsters, Arrays of heated horizontal cylinders in natural convection, Thermosciences Report 3–71, Department of Mechanical Engineering, Queen's University, Kingston, Canada (1971).
12. H. C. Hottel and A. F. Sarofim, *Radiation Transfer*. McGraw-Hill, New York (1967).

APPENDIX

As indicated earlier, the effect of radiant heat exchange is of considerable importance in the results presented here. In the first place, the emissivity of the surface of the heated elements has not been determined. The estimated values used, while thought to be satisfactory, certainly contribute to the uncertainty in the results reported here.

A second complication arises from the exchange of heat via radiation between adjacent cylinders in the array. Unlike the case of a single heated element exchanging heat with surroundings at uniform temperature, there exists no simple expression for the radiant heat transfer coefficient for multiple element arrays. For a single element, the well-known result for the radiative surface coefficient, h_r, is:

$$h_r = \varepsilon\sigma(T_s^2 + T_\infty^2)(T_s + T_\infty). \qquad \text{(A.1)}$$

In this equation, T_s is the temperature of the surface, ε the surface emissivity and T_∞ the temperature of the surrounding fluid. It is assumed that the enclosure surfaces are also at T_∞. Then,

$$Q_r = h_r A(T_s - T_\infty). \qquad \text{(A.2)}$$

The total heat transfer to the surroundings is the sum of Q_r and the convective heat transfer, Q_c. In an experiment of the type reported here this is just equal to the Joule heating, IV, provided that only one heated cylinder is used.

In the case of multiple elements, equation (A.2) is no longer satisfactory, because the surroundings of any cylinder now consist of the unheated enclosure plus the neighbouring heated surfaces. The problem can be handled by the method of Gebhart [10], but the computations are fairly tedious. Therefore it seems desirable to seek an approximate method.

The approximate method developed to reduce the data for the arrays is discussed fully in [11]. The essence of the approximation is as follows. An "interior" cylinder in an array "sees" only the two adjacent cylinders and the surroundings. The angle factor between long parallel cylinders is given by [12].

$$F_{12} = \frac{1}{\pi}\left\{\sin^{-1}\left(\frac{1}{S}\right) + \sqrt{(S^2 - 1)} - S\right\} \qquad \text{(A.3)}$$

where S is the spacing in cylinder diameters. It may be shown that the direct radiant exchange between cylinders is negligibly small, and it is assumed that neighbouring cylinders are in radiant balance. In this event, the only net radiant exchange is with the enclosure walls. Thus, it is reasonable to assume that the radiant heat exchange with the enclosure walls is the same as that for a single cylinder *but with a correction for the reduced effective angle factor*. This correction may be absorbed into a new surface coefficient, h_r', incorporated into an approximate radiative expression.

$$Q_{r,a} = h_r' A(T_{s_i} - T_\infty) \quad \text{(A.4)}$$

where

$$h_r' = h_r(1 - K F_{i,i+j}).$$

In this last expression K is unity, if the cylinder is at the edge of the array, or two, if the cylinder is in the interior of the array.

While it is clear that the approximation suggested is satisfactory for large spacing, it is not recommended for use for small spacing. The radiative transfer has been evaluated using the approximate method and compared with that obtained using the absorption factor method. For an interior cylinder at a total dissipation rate of $3{\cdot}08 \times 10^3$ W/m^2, the radiation component is calculated to be 1·4 per cent of the total dissipation using the approximate expression and 1·13 per cent of the total using the absorption factor method. These figures refer to the most severe case for which data are reported. Because of the inaccuracy in the value of surface emissivity, the use of the approximate method of calculating the effective radiation transfer is entirely justified.

CONVECTION NATURELLE AUTOUR D'UN ALIGNEMENT DE CYLINDRES HORIZONTAUX CHAUFFÉS

Résumé—Les propriétés du transfert thermique d'un alignement vertical de cylindres chauffés sont obtenues dans le cas de la convection naturelle permanente. On présente des résultats pour une variété d'espacements et de nombres de cylindres. Pour chacune de ces combinaisons on a réalisé différents flux de chaleur qui correspondent à des nombres de Grashof compris entre 750 et 2000 pour le cylindre. Les résultats sont présentés en fonction de l'excès de température de la surface du cylindre inférieur. Comme dans le cas des études de Lieberman et Gebhart [2], la température superficielle croît avec l'élévation dans le dispositif pour un pas réduit, mais elle décroît avec l'élévation quand l'espacement est suffisamment grand. Ce comportement s'explique à partir des champs de vitesse et de température dans le sillage d'une source linéaire. On montre que l'accord entre les résultats présents et ceux de la référence 2 est satisfaisant quand la longueur correcte est utilisée dans le nombre de Grashof. A cause de l'interaction des cylindres, les nombres de Nusselt de cylindre ne sont pas évalués par les formules usuelles de convection naturelle. Les nombres effectifs de Nusselt de cylindre sont comparés à ceux prédits en convection naturelle simple pour le nombre de Rayleigh du cylindre. Leur rapport est représenté graphiquement en fonction de la position dans le réseau. Les nombres moyens de Nusselt pour les alignements sont donnés en fonction du nombre de Rayleigh.

FREIE KONVEKTION BEI ANORDNUNGEN BEHEIZTER, HORIZONTALER ZYLINDER

Zusammenfassung—Die Wärmeübertragungseigenschaften einer vertikalen Anordnung beheizter Zylinder werden für die stationäre freie Konvektion bestimmt. Ergebnisse wurden erhalten für eine Auswahl von Anordnungen mit variablem gegenseitigen Abstand und variabler Zahl der Zylinder. Für jede dieser Anordnungen wurden verschiedene Dissipationsraten aufgebracht, wobei sich Grashof-Zahlen von 750–2000 ergaben. Die Ergebnisse werden als Übertemperaturen der Zylinderoberfläche dargestellt, bezogen auf die Übertemperatur des untersten Zylinders. Wie auch bei den Untersuchungen von Liebermann und Gebhart [2], steigt bei geringem Abstand der Zylinder die Oberflächentemperatur mit der Höhe der Anordnung, sie sinkt jedoch, wenn der Abstand der Zylinder hinreichend gross ist. Dieses Verhalten wird an den Geschwindigkeits-und Temperaturfeldern im Nachstrom einer Linienquelle erklärt. Es wird gezeigt, dass diese Ergebnisse mit den Ergebnissen aus [2] übereinstimmen, wenn in der Grashof-Zahl die richtige Länge eingesetzt wird. Aufgrund der gegenseitigen Wechselwirkung der Zylinder können die Nusselt-Zahlen für Zylinder nicht durch die üblichen Beziehungen der freien Konvektion bestimmt werden. Die tatsächlichen Nusselt-Zahlen werden mit Nusselt-Zahlen verglichen, die für einfache, freie Konvektion mit der Rayleigh-Zahl für Zylinder bestimmt wurden. Diese Verhältnisse sind über der Ortskoordinate der Anordnung aufgetragen. Mittlere Nusselt-Zahlen für die Anordnungen sind als Funktion der Rayleigh-Zahlen ebenfalls aufgetragen.

РЯДЫ НАГРЕТЫХ ГОРИЗОНТАЛЬНЫХ ЦИЛИНДРОВ В УСЛОВИЯХ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация—Получены характеристики теплопереноса вертикального ряда нагретых цилиндров в стационарных условиях естественной конвекции. Представлены результаты экспериментов с различными вариантами расстояния между цилиндрами и разным

количеством цилиндров. Для каждого из вариантов использовались различные скорости диссипации, в результате чего числа Грасгофа для цилиндра изменялись от 750 до 2000. Результаты выражались через избыточную температуру поверхности цилиндра, нормализовнную избытоуной температурой самого нижнего цилиндра. Как и в работах Либермана и Гебхарта [2] температура поверхности увеличивается по мере поднятия по ряду для рядов с малым расстоянием между цилиндрами и уменьшается с подъемом по ряду, если расстояние между цилиндрами достаточно большое. Это объясняется полями скорости и температурными полями в следе линейного источника. Полученные результаты удовлетворительно согласуются с данными других авторов [2], если правильно выбран характерный размер для определения чисел Грасгофа. В силу взаимодействия цилиндров числа Нуссельта не могут быть рассчитаны с помощью обычных соотношений естественной конвекции. Действительные числа Нуссельта сравниваются с числами, рассчитанными для случая простой естественной конвекции при соответствующем числе Релея. Построены графики для этого отношения в зависимости от положения в ряду, а также для усредненных чисел Нуссельта в зависимости от числа Релея.